About polarization and Poincaré sphere

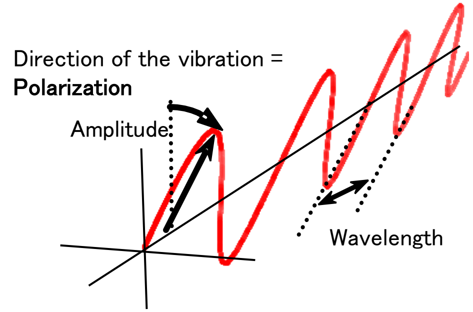
1. About polarization and birefringence
2. The Poincaré sphere

1． About polarization and birefringence

About polarization

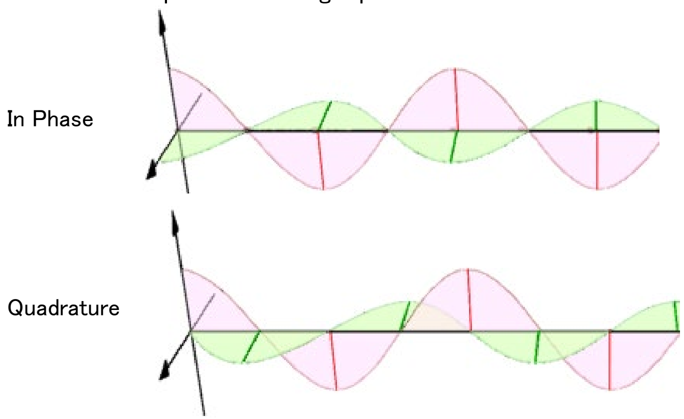
◆ Polarization = Vibration direction

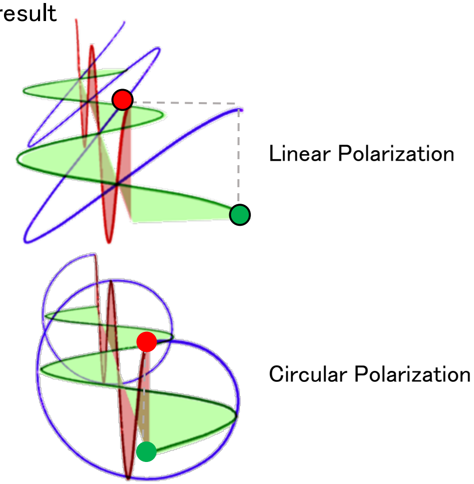
Amplitude (= brightness), wavelength (= color) and vibration direction (= polarization) are the three basic properties of light as an electro-magnetic wave.
Polarization cannot be perceived by the naked eye directly. This fact makes an intuitive understanding of polarization difficult. However, polarization is utilized in a wide range of applications, like LCD monitors, to mention only one of them.



◆ Phase Shift of each Polarization Component and Total Polarization Resulting

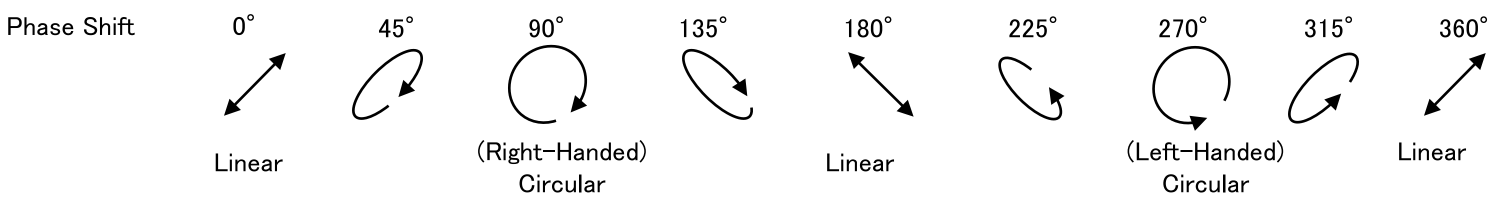
A state of polarization can be either linear (vibration occurs along a straight line), circular, elliptical or any intermediate state between those. Each single state of polarization can be understood as the result of a combination of 2 perpendicular components having a phase shift between them.





The amount of phase difference between the two components can be expressed either as an angle or as a distance, as in the relation below:
(distance) 1 wavelength = (angle) 360 degrees = 2 π (radian)

※ In the case of other phase differences → Elliptical polarization



About birefringence, phase difference and internal stress

◆ Birefringence as a Characteristic of materials

◇ About birefringence and the photo-elastic coefficient

When a force is applied to a structure, internal stress occurs inside the material, result in what is called birefringence. Birefringence is the property of a material that introduces a phase shift between the two perpendicular components of light polarization.
This ability to generate a phase shift in reaction to applied stress is different for each material. It is typically big is plastic-like materials like polycarbonates, and small in glass-like materials. The ability to generate birefringence is called the "photo-elastic" constant.

Materials	Photo-elastic constant (10^{-12}/Pa)
Quartz	3.5
Polycarbonate	75
Acrylic	6
Glass	0.5
Lead glass	0.005

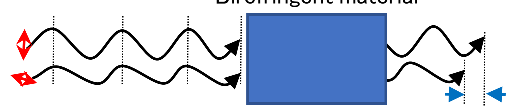
phase shift δ (nm) = β × thickness d (cm) × stress [force] F (10^5 Pa)
β is the photo-elastic constant of the material (10^{12}/Pa)]

For example, a force of 1MPa applied to a 1mm quartz plate results in a phase shift of:

$$3.5 \times 0.1 \times 10 = 3.5\text{nm}$$

◇Birefringence, phase shift and polarization

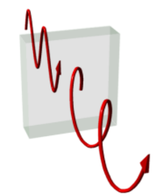
Introducing a phase-shift between the two components of polarization is equivalent as saying that the refractive index felt by each components is different, thus the name "bi-refringence".
Because of the change of phase between polarization components, the state of polarization of the light going through the material is modified. In means that, on the contrary, one can measure the phase shift introduced by the material by measuring the change of polarization of light going through it. The phase-shift information can then be used, for example, to evaluate internal stress applied in the material, using the relations introduced above.



【 non birefringent material 】

No change in polarization

【 birefringent material 】

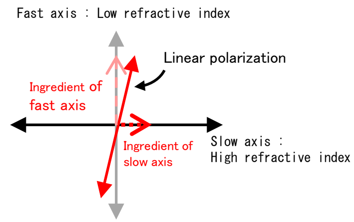


Change of polarization

◇Polarization change is dependent on birefringence axis

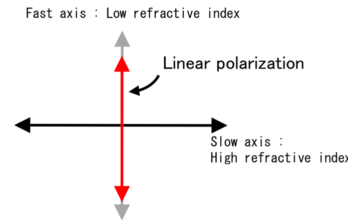
The change of polarization depends on the phase shift, but also on the axis orientation of birefringence, i.e. the direction in which input polarization is decomposed in two perpendicular components.

For example, linear polarization may be unchanged if birefringence axis is aligned, or perpendicular, to its direction. On the other hand, circular polarization will always be affected by birefringence, whatever angle birefringence axis is oriented at.



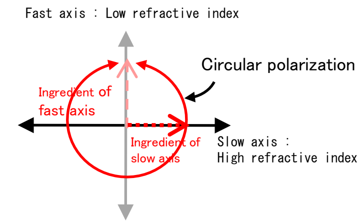
Axis of linear polarization is not equal to fast axis or slow axis

Polarization change depending on a phase shift occurs, because of the introduction of a phase shift between fast or slow axis components.



Axis of linear polarization is equal to fast axis or slow axis

Only one component is present, so no phase shift can be introduced, regardless of the phase-shift value.



Circular polarization

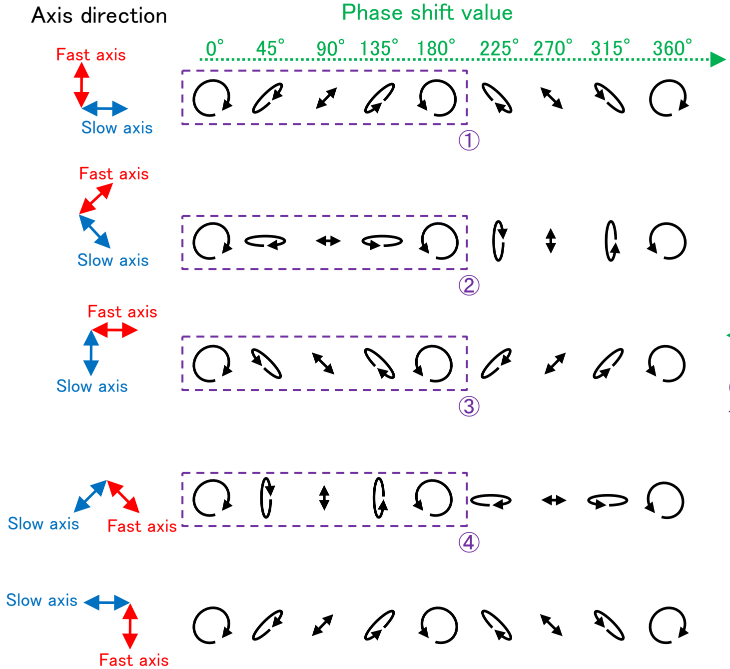
Circular polarization is always affected by the phase shift, as every direction is equivalent when dividing light into two orthogonal components.

2. The Poincaré sphere

The Poincaré sphere

◆ Change of polarization induced by transmission through a material introducing a phase shift.

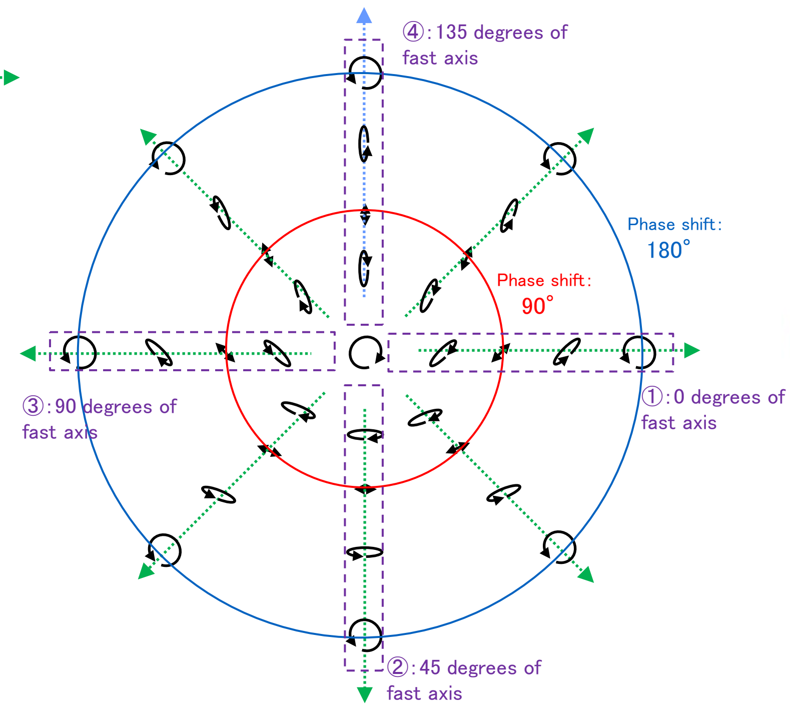
For any phase shift value and axis direction.



The state of polarization returns to its original state when shifted by 360-degree, for every possible direction.
Axis directions of 0 and 180 degrees are equivalent regarding the result on output polarization.

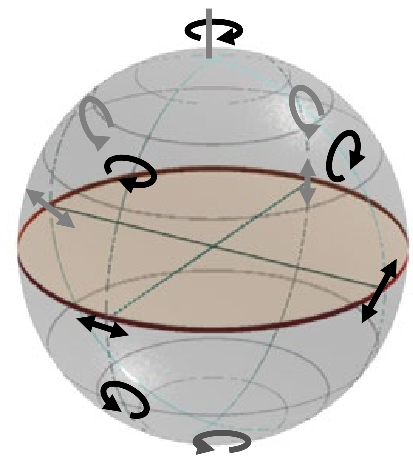
◆ Graphics representation

Equal phase shifts are represented on the same circle. The position on the circle is determined by axis orientation.



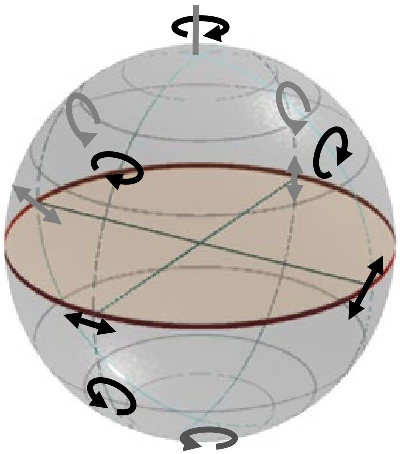
◆ In spherical coordinates

Mapped into spherical coordinates, the graphical representation of the state of polarization is called the Poincaré sphere.



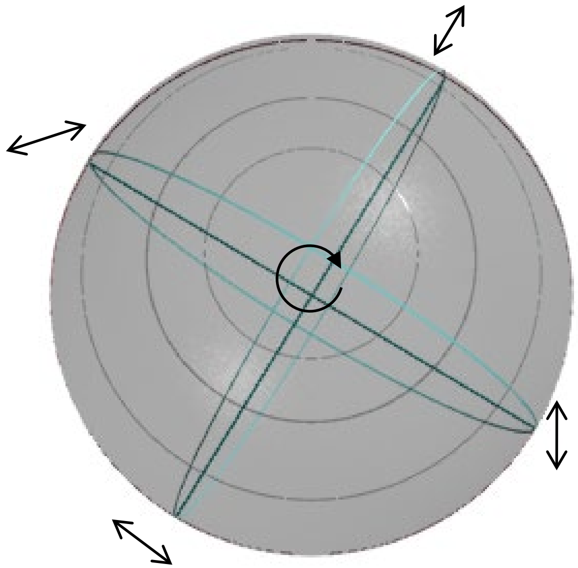
The Poincaré sphere displays each possible state of polarization as a point in a spherical coordinate system.

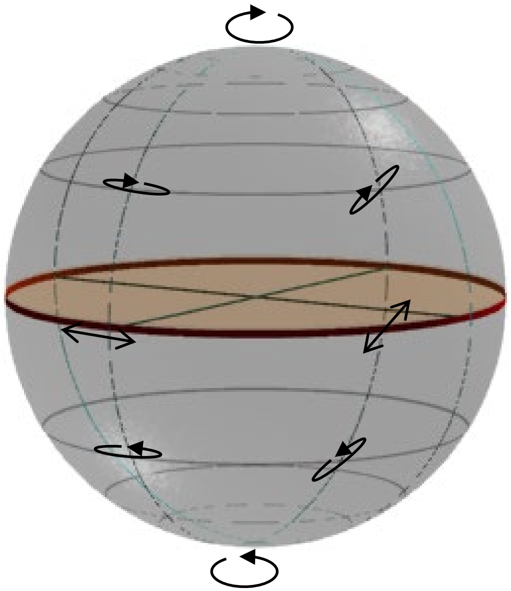
Basic rules on the Poincaré sphere



Linear polarizations are placed on the equator.
Two perpendicular linear polarizations face each other across the center

States of polarization laying on the same line of longitude has the same axis direction.
The ellipticity varies with latitude, being maximal for circular polarization at the poles.





Convenient usage of the Poincaré sphere

Polarization change after the transmission through a birefringent medium can be understood intuitively by a trajectory on the Poincaré sphere.

【 Polarization change introduced by birefringence】

The change of polarization after the transmission is expressed as a movement on the sphere following the rules below:

① Define a rotation axis lying in the equatorial plane, whose direction reflects the birefringence axis orientation. (cf. right figure)

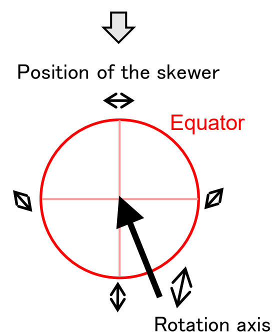
② Rotate the point expressing input polarization around the rotation axis defined above, of an amount equal to the birefringence induced phase shift.

Phase shift	Angle of rotation / degree
λ	360
λ/2	180
λ/4	90

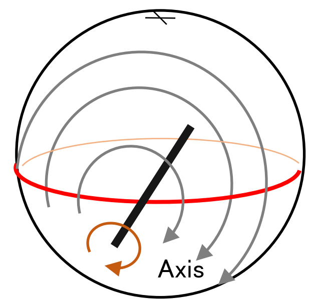
③ Output polarization is what is expressed by the point obtained after the rotation applied above.

① Position of the rotation axis

Example for phase shift direction =

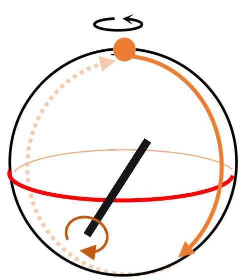


② Rotation proportional to phase shift



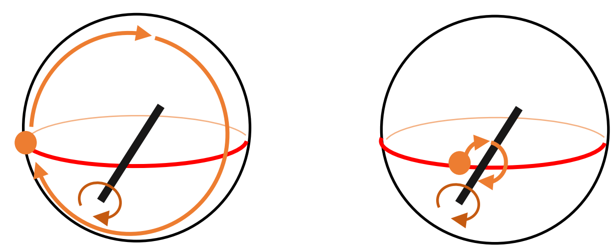
ex1. Change from circular polarization

Circular polarization → Linear polarization → Circular polarization



ex2. Change from linear polarization

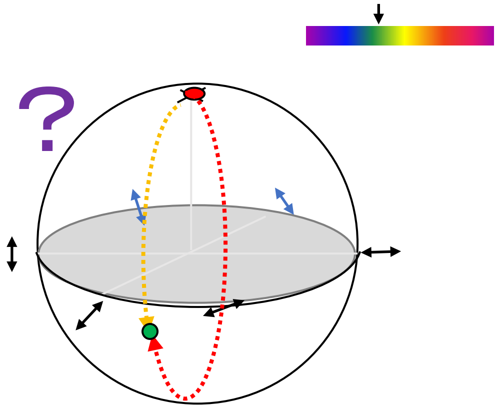
The change of polarization is smaller when input polarization axis and birefringence axis are close.



Wide range evaluation using WPA series explained on the Poincaré sphere

PA/WPA systems use circular polarization for input light, which is equivalent to put the starting point to the pole on the Poincare sphere. After transmission through the sample to be evaluated, states of polarization can be plotted on the Poincare sphere the phase shift computed.

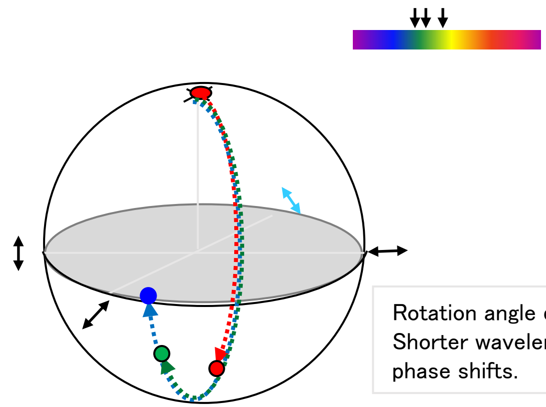
【Single-wavelength measurement mode】



In single-wavelength measurement, we cannot distinguish a given rotation from its opposite rotation (corresponding to birefringence axis at right angle) when phase shift is greater than λ/2.
(In the figure, both rotations shown as a red dotted line and as a yellow dotted line rotations lead to the same green point.)
Therefore, the upper limit of the measurement is λ/2.
As default the angle of rotation leading to the smallest rotation (= phase shift) is chosen. (yellow dotted line in the figure.)

→ Measurement range < λ/2

【 Triple-wavelength measurement mode 】



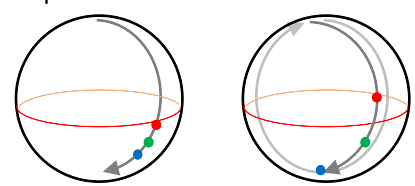
Rotation angle depends on wavelength.
Shorter wavelengths tends to show larger phase shifts.

Using the dispersion of points measured at 3 different wavelengths. The real direction can be judged even when the phase shift is greater than direction where points form a line, the rotatory direction is judged λ/2.

→ Measurement range > λ/2

Furthermore, even when the phase shift is greater than 1 wavelength – one or more "laps" around the Poincaré sphere – the distance between the 3 points can be used to evaluate the actual number of laps. It can estimate the number of the laps by the expanse of the point of 3 wavelengths (see the drawing below).

→ Measurement range > λ



First lap Second lap

The upper limit in triple-wavelength measurement mode is bigger than measurement wavelength.
It can be easily understood on the Poincare sphere.

To the question “How does this birefringence modifies this polarization?”
Just use the Poincaré sphere!

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